On population abundance and species niche relationship during invasion

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Abstract

We studied the the effect of heterogeneous environment in the relationship between population abundance and range dynamics by exploring two models: I) a model where the Allee effect is constant and independent of environmental conditions (niche conditions) and 2) a model where the Allee intensity depends on niche conditions. Results show that the long term species distribution patterns are independent from the way that the Allee effect its included in the logistic growth model. However we observed that population abundance patterns in the short term dynamics are very sensitive to the considered model. This fact can have important consequences in the prediction of how the invasion process occurs in nature, for example in the estimation of the arriving time of an invasive species to a certain location.

Introduction

Invasive species are one of the most important problems worldwide because of their impact on biodiversity, in ecosystem dynamics and in the services they provide. Hence it is important to study them both theoretically and empirically. Regarding to the theoretical aspect certain models have been proposed that consider factors related to the biology of the species *per se*, this is the case of Allee effects, which depending on its intensity can prevent or not the colonization process [3, 2]. A matter of interest is also studying under which environmental conditions an invasion will be successful. In this work we show a way to include the effect of heterogeneous environment on invasion dynamics under the context ecological niche theory *sensu* Grinnell [1, 5]. Also we compare the range dynamics of this kind of model with one where the Allee intensity is independent of environmental conditions.

Objective

To study the effect of heterogeneous environment in the range dynamics and population abundance of a species that invades a geographical area.

General Framework

Let assume that the fundamental niche of species j is measured in terms of habitat suitability S^j and it in turn depends on environmental conditions E in the following form (*see* [4]):

1. Let E a vector of environmental variables such that:

$$E = \{(e_1, e_2, \cdots, e_n)\,| e_1^{min} \le e_1 \le e_1^{max}; e_2^{min} \le e_2 \le e_2^{max}; ...; e_{min}^n \le e_n \le e_{max}^n\} \quad (1)$$

2. The suitability function S^j of species j with $j = 1, \cdot, m$, is a function of vector E and it is defined in the closed interval $[0, 1]$:

$$S^j : E \to [0, 1] \quad (2)$$

3. Finally the intrinsic growth rate r^j of species j is a function that depends on S^j

$$[0, 1] \xrightarrow{r^j} \mathbb{R}^+ \quad (3)$$

The models

We consider two types of invasion models: 1) one where the Allee effect is constant throughout a geographical area and 2) one where Allee intensity depends on the environmental conditions (niche conditions).

Model 1: constant Allee model

$$\dot{x}_i = r_i x_i \left(1 - \frac{x_i}{K_i}\right)(x_i - \mathrm{M}) + \sum_h \delta_{h\,i} x_h - \sum_h \delta'_{i\,h} x_i, \quad (4)$$

Model 2: Environmental dependent Allee

$$\dot{x}_i = r_i x_i \left(1 - \frac{x_i}{K_i}\right)(x_i - \mathrm{M}_i) + \sum_h \delta_{h\,i} x_h - \sum_h \delta'_{i\,h} x_i, \quad (5)$$

where $\dot{x}_i$ is the rate of change of a population, growing in the cell i at time t. r_i is the intrinsic growth rate of a population in the cell i. We considered r_i to be a lineal function of r, the intrinsic growth rate at optimal conditions and the suitability s_i ($r_i = r(s_i)$). $K_i = K\,r_i$ is the carrying capacity of cell i. $\delta_{h\,i}$ is the immigration rate of individuals coming from others cells to the i cell. Finally $\delta'_{i\,h}$ is the emigration. M is the Allee effect; for model 5, $\mathrm{M}_i = \frac{\mathrm{M}}{r_i}$.

Species niche and Geographic area

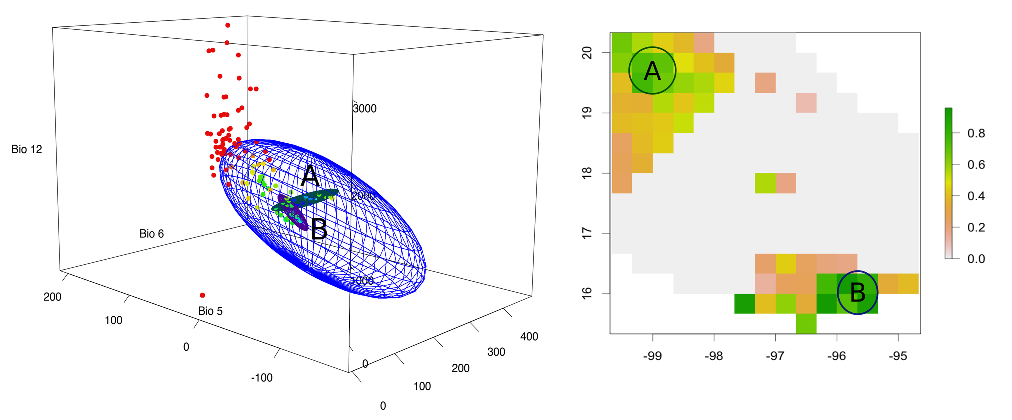


Figure 1: The fundamental niche and suitability map for the virtual species. Left panel: the blue ellipsoid represents the fundamental niche of the virtual species, with red points outside the niche, and yellow, orange, green and blue, cells inside the niche. Right panel: the corresponding cells in geographic space, with greener colors showing higher suitability. A and B denote the same set of cells in both niche and geographic spaces.

Model Parametrization

Dispersal kernel

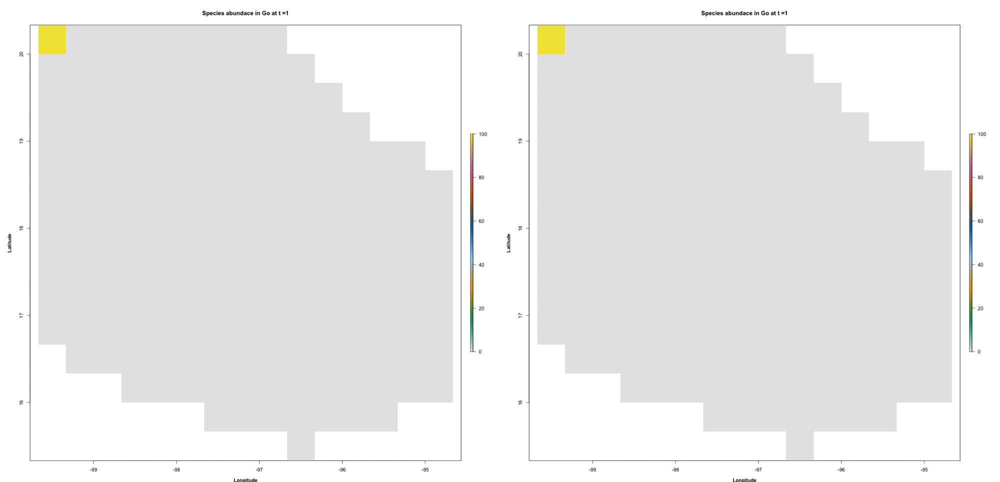
$$\delta_{i\,h}(w_{i\,h}) = \begin{cases} a \exp\left[-w_{ih}^b\right] & w \le D_{max} \quad (6a) \\ 0 & w > D_{max} \quad (6b) \end{cases}$$

$r = 0.15$; $\mathrm{M} = 8$; $K = 10001$; $a = 0.002$; $b = 7$; $D_{max} = 2$

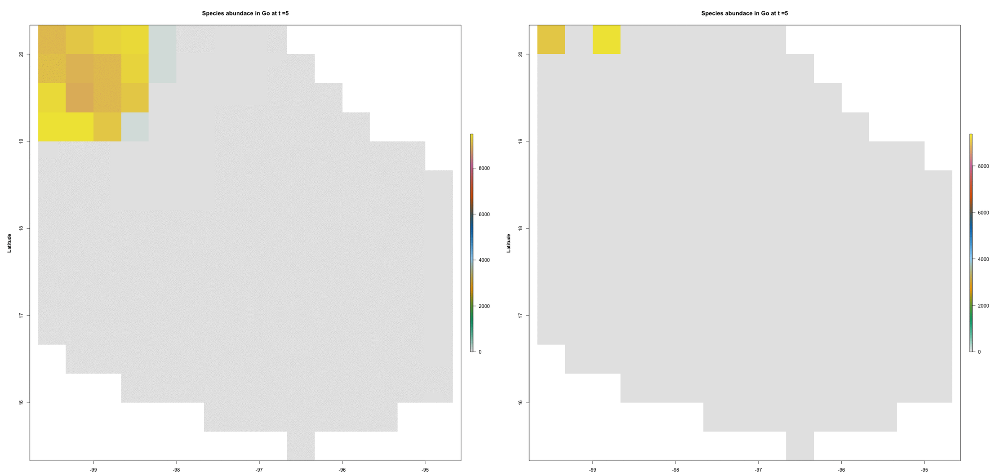
Results

Constant Allee **Environmental dependent**

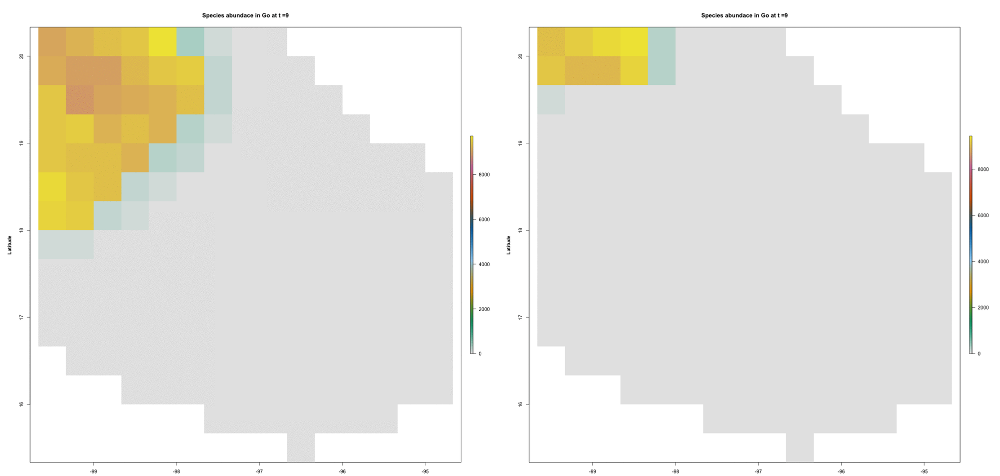
(A) Short term dynamics at t = 0



(B) Short term dynamics at t = 5



(C) Short term dynamics at t = 9



(D) Long term dynamics at t = 101

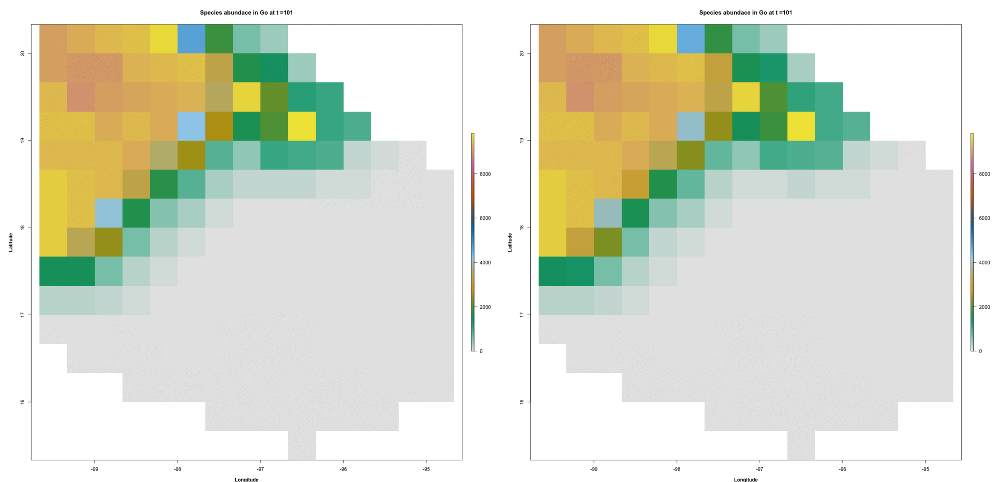


Figure 2: Geographic abundance distribution at different times (short term $t = 0$, $t = 5$, $t = 9$ and long term dynamics $t = 101$) for model 4 (left panel) and 5 (right panel).

Abundance distribution for the constant Allee effect model

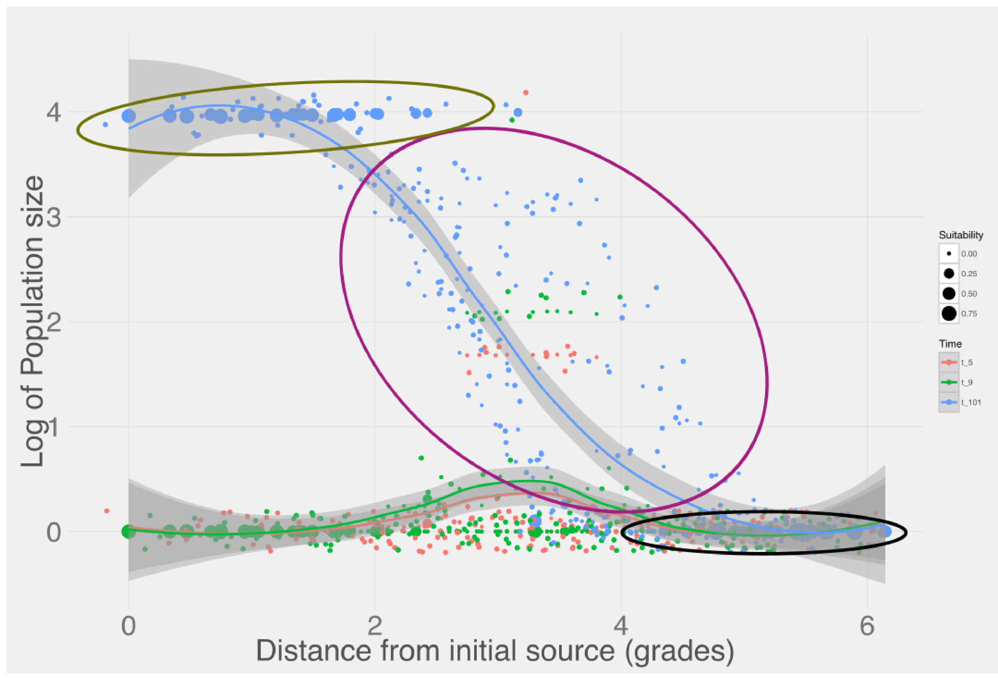


Figure 3: Abundance distribution for the constant Allee effect model, in terms of the distances from the initial invasion source and the habitat suitability (S_i), for different time periods (short term $t = 5$ − red points; $t = 9$ − green points and long term dynamics $t = 101$ − blue points). Each oval bounds a region with similar suitability conditions: high suitability conditions are found in the green oval and also in the black one. For short term, there is no apparent pattern.

Conclusions

1. Abundance and geographic distribution patterns are very sensitive to the heterogeneous environmental conditions for short term dynamics.
2. For long term dynamics it appears that the abundance and distribution patterns does not change between both models.
3. For the constant Allee model we found a negative relationship between distance from initial invasion source and abundance which in turn is modulated by habitat suitability.

References

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